Chapter 7

The Effect of Splitting Objectives in the Analytic Hierarchy Process

7.1 Introduction

In any multi-attribute decision making (MADM) problem, structuring of attributes is the most important task. An ill-structured problem can lead to a wrong decision. Weber et al. (1988, page 431) writes:

"The structure of objectives not only defines the scope and the detail of evaluative considerations, but it also is likely to shape the numerical inputs that are elicited in later stages of the evaluation."

Substantial work has been done on the structure of objectives in multi-attribute utility measurement model. Green and Srinivasan (1978) and Hauser and Urban (1977) have shown how the details of attribute structures influence people's evaluations of products in marketing. Edwards (1977) studied the consequences of detailed attribute specification in social program evaluation. The same problem had been investigated in water resources planning by Keeney and Wood (1977) and Gershon and Duckstein (1980). Kleinmuntz (1983), in his simulation study, showed that omission of attributes can lead to substantial reductions in model validities. Weber et al. (1988) examined how weights in multi-attribute utility measurement change when objectives are splitted into more detailed levels. In their experiment on a job selection problem, each of the objectives, namely, job security, income, and career opportunities, was subdivided into two sub-objectives, called attributes, and people were asked to weight them by four weight determination techniques, namely, Ratio, Otto, Swing, and Conjoint. Their robust finding was that the more detailed parts of the value tree were weighted significantly higher than the less detailed ones. In a separate but related context, Fischer et al. (1987) showed that assessing utility functions over proxy attributes requires complex inferences which may exceed the human capacity for consistent judgment, thus biasing utility assessments.

Virtually, no work has been done on how the criteria weights are affected when criteria are splitted into more detailed ones in AHP model. In order to fill up the gap, the present experiment addresses the problem by varying the specificity with which an attribute is defined and determining the effect of the attribute details on the global weights of the alternatives.

7.2 General Description of the Problem

The present problem can be posed either as an investigation of the effect of splitting objectives on the global weights of alternatives, or as an experimental verification of the fourth axiom of AHP. (Fourth axiom of AHP: All criteria and alternatives are represented in the hierarchy (Saaty, 1986a)).

The experiment:

Three higher secondary school of Calcutta, namely, South Point School (X), Hindu School (Y), and Hare School (Z) are primarily selected for admission purpose. The best one should be chosen out of these three schools. The selection is based on several criteria. Initially we consider three gross criteria (called objectives), namely, education (E), vocational training (V), and extra-curricular activities (EA). The weights of these objectives are calculated by AHP. Later, each objective is splitted at a time into two attributes ('education' into 'teaching' (T) and 'academic environment' (A); 'vocational training' into 'facility of vocational training' (F) and 'quality of vocational training' (Q); extra-curricular activities' into 'sports' (S) and 'cultural activities' (C)) and their corresponding weights are calculated. In the second stage, two objectives from the three are splitted at a time, and their weights are computed by using the AHP. Finally, all the three objectives are splitted at a time and their weights are determined. The hierarchy is shown in Fig. 7.1.

Subjects:

Thirty-six people from six categories (each category consist of six persons) of profession served as subjects. The categories are: 1) faculty members, 2) sponsored research scholars, 3) non-teaching staff, and 4) post-graduate students of Indian Institute of Technology (IIT), Kharagpur, 5) self entrepreneurs, and 6) persons engaged in sports and cultural activities in an around IIT campus.

The judgmental preferences are elicited from the subjects on individual contact basis. After introducing the problem, we emphasized on the explanation of the (1/9-9) ratio scale and the semantic interpretation of each ratio on this scale. Then we presented the questionnaire. The form of the questions is *'between two factors which one is more important and how much more?'*. They were asked to give their judgments taking point estimates from the (1/9-9) scale.

7.3 AHP and Additive Value Function

The measure of preference obtained by applying AHP to multicriteria decision making problems under certainty satisfies the definition of an additive value function. Consequently, the AHP can only be applied to multicriteria decision making problems in which the conditions for the existence of an additive value function are satisfied.

Definition 7.1: A function V, which associates a real number V(x) to each point (or attribute) x in an evaluation space, is said to be a value function representing the decision maker's preference structure provided that

$$x' \sim x'' \Leftrightarrow V(x') = V(x'')$$

$$\text{and} \quad x' \succ x'' \Leftrightarrow V(x') > V(x'')$$

where $x' \sim x''$ and $x' \succ x''$ are to be read as x' is indifferent to x'' and x' is preferred to x'', respectively.

Definition 7.2: A preference structure is additive if there exists a value function reflecting that preference structure, which can be expressed by

$$V(x,y) = V_1(x) + V_2(y),$$

where $V(x,y)$ is the joint value function of the attributes x and y; $V_1(x)$ and $V_2(y)$ are the value functions of the single variables x and y, respectively. If the preferences of the decision maker can be represented by an additive value function V', then

$$V'(x_{i1}, x_{i2}, \ldots, x_{in}) = \sum_{j=1}^{n} V'_j(x_{ij}) \tag{7.1}$$

where V'_j denote the value function of a single attribute C_j. It can be shown (Keeney and Raiffa, 1976) that if V' is bounded, then Equation (7.1) can be rewritten as

$$V(x_{i1}, x_{i2}, \ldots, x_{in}) = \sum_{j=1}^{n} w_j V_j(x_{ij}) \tag{7.2}$$

where V and V_j, j = 1, 2, …, n, are scaled from zero to one and

$$\sum_{j=1}^{n} w_j = 1 \tag{7.3}$$

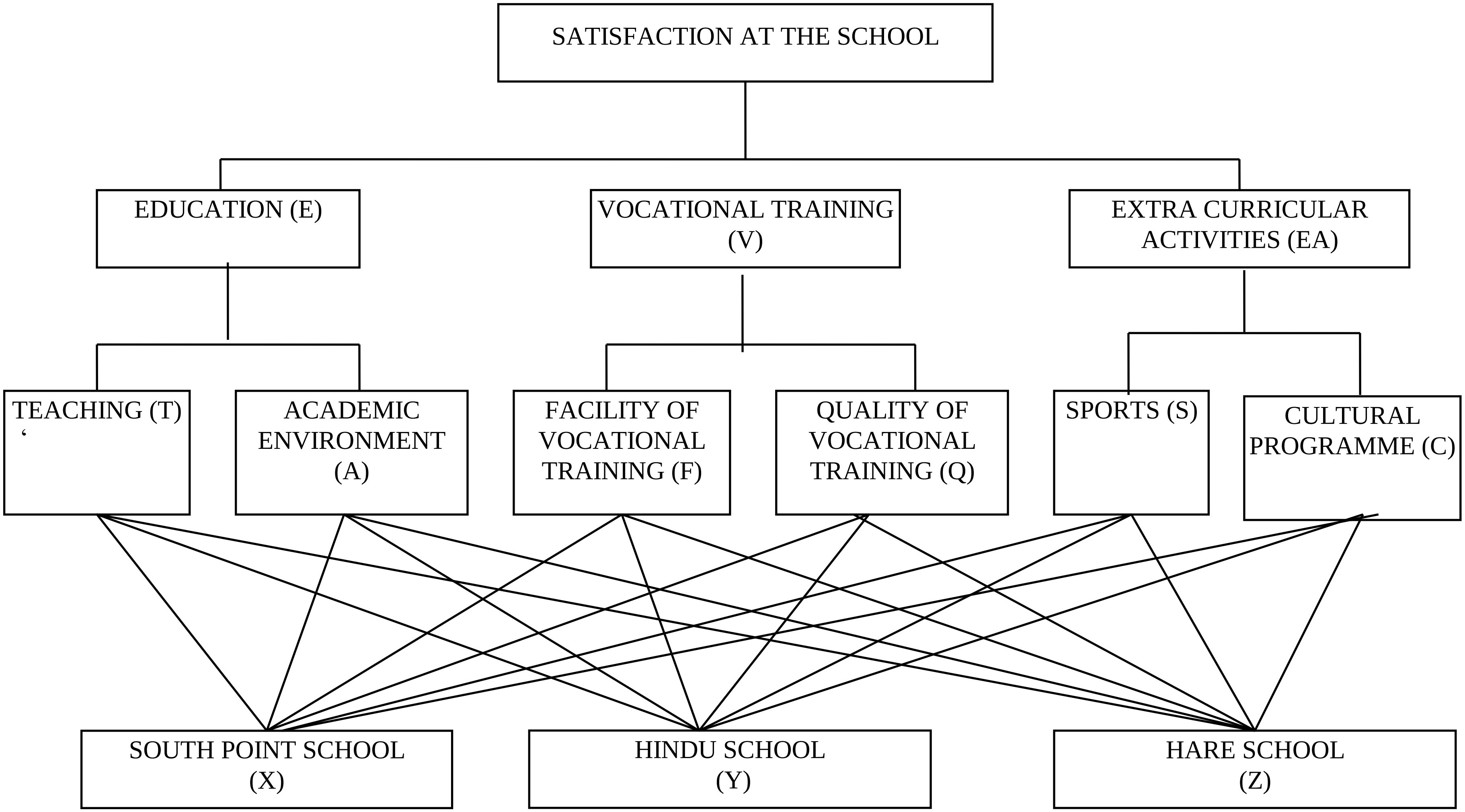

FIG 7.1: HIERARCHY OF THE SCHOOL SELECTION PROBLEM.

In AHP, the global weight of alternative A_i for all the criteria taken into account simultaneously, is given by

$$r_i = \sum_{j=1}^{n} w_j u_{ij} \qquad (4)$$

where w_j and u_{ij} denote, respectively, the weights of the jth criterion and the local weight of alternative A_i with respect to the jth criterion. That alternative is chosen which maximizes r_i, i = 1,2, …, n.

In view of Equations (7.2) and (7.4), it can be said that r_i represents valuation of an additive function. Thus

the necessary condition for the existence of an additive value function, namely, the attributes be mutually preferentially independent, is also necessary for a rigorous application of the AHP.

It is well known in AHP theory that in order to use Equation (7.4), the attributes should be mutually preferentially independent.

7.4 Effect of Splitting Objectives on the Weights of the Objectives

In the first experiment, the weights of the three objectives have been elicited. Then the weights of the four attributes (splitting one objective into two attributes) have been calculated. Incorporation of the full list of answers by the respondents (i.e., all the 288 comparison matrices out of which the number of 3 × 3, 4 × 4, 5 × 5, and 6 × 6 matrices are 36, 108, 108, and 36, respectively) is not possible for obvious reason. The pairwise comparison matrices for only one person are shown in Table 7.1. The weights of the objectives (when one objective is splitted into two attributes) are shown in Table 7.2. We have presented only six persons judgments taking one from each category.

Table 7.3 shows the weights of 5 attributes (splitting two objectives into two attributes each). The weights of the six attributes (splitting all the objectives into two attributes each) are also shown in Table 7.3.

7.4.1 Discussion: Table 7.2 (Case of splitting single objective)

The number in the middle of two rows is the sum of the two elements (in the left side) immediately above and below it. It is to be noted that the sum of the weights for the detailed attributes is always greater than the weight directly attached to the objective that is detailed by these attributes. Further, the weight of the unsplitted objectives are decreased. For example, let us take the judgment of the fifth person. The weights of the three objectives, education, vocational training and extra-curricular activities are 0.6833, 0.1168, and 0.1998, respectively. Next, each objective is subdivided at a time into two attributes. The weights of the three objectives in 3 splitting cases are: (0.8177, 0.0754, 0.1069), (0.5673, 0.3579, 0.0747) and (0.06060, 0.1338, 0.2602). In the first case, the weight 0.8177 of the objective education has been increased substantially from 0.6833. The same is true for splitting of the objectives vocational training and extra-curricular activities.

Table 7.1: Pairwise comparison matrices of attributes for Person 1

	E	V	EA
E	1	8	5
V	1/8	1	1/6
EA	1/5	6	1
Weights:	0.7188	0.0579	0.2234

	T	A	V	EA
T	1	1	7	4
A	1	1	7	3
V	1/7	1/7	1	1/6
EA	1/4	1/3	6	1
Weights:	0.4180	0.3801	0.0444	0.1575

	E	F	Q	EA
E	1	6	7	5
F	1/6	1	3	1/3
Q	1/7	1/3	1	1/4
EA	1/5	3	4	1
Weights:	0.6342	0.1053	0.0550	0.2056

	E	V	S	C
E	1	8	6	5
V	1/8	1	1/3	1/4
S	1/6	3	1	1/2
C	1/5	4	2	1
Weights:	0.6493	0.0529	0.1149	0.1830

Table 7.2: Results of splitting one objective

Person	Att	Weights	Att	Weights		Att	Weights		At	Weights	
1)	E	0.7188	T	0.4180		E	0.6342		E	0.6493	
					0.798 1						
	V	0.0579	A	0.3801		F	0.1053		V	0.0529	
								0.160 3			
	EA	0.2234	V	0.0444		Q	0.5550		S	0.1149	
											0.297 9
			EA	0.1575		EA	0.2056		C	0.1830	
2)	E	0.7306	T	0.5031		E	0.6658		E	0.6347	
					0.830 4						
	V	0.0810	A	0.3273		F	0.0501		V	0.0624	
								0.169 4			
	EA	0.1884	V	0.0530		Q	0.1193		S	0.1514	
											0.302 8
			EA	0.1105		EA	0.1648		C	0.1514	
3)	E	0.6738	T	0.4138		E	0.6295		E	0.5674	
					0.793 3						
	V	0.2255	A	0.3795		F	0.1156		V	0.2822	
								0.284 7			
	EA	0.1007	V	0.1398		Q	0.1691		S	0.1089	
											0.150 3
			EA	0.0668		EA	0.0858		C	0.0414	
4)	E	0.1642	T	0.0622		E	0.0902		E	0.2763	
					0.225 2						
	V	0.6569	A	0.1630		F	0.4753		V	0.4873	
								0.784 1			
	EA	0.1963	V	0.5501		Q	0.3088		S	0.1182	
											0.236 4
			EA	0.2247		EA	0.1257		C	0.1182	
5)	E	0.6833	T	0.3172		E	0.5673		E	0.6060	
					0.817 7						
	V	0.1168	A	0.5005		F	0.1547		V	0.1338	
								0.357 9			
	EA	0.1998	V	0.0754		Q	0.2032		S	0.1913	
											0.260 2
			EA	0.1069		EA	0.0747		C	0.0689	
6)	E	0.7440	T	0.6139		E	0.6513		E	0.5870	
					0.759						

Atts	Wts	Atts	Wts		Atts	Wts		Atts	Wts	
				4						
V	0.1336	A	0.1455		F	0.1193		V	0.1712	
							0.2386			
EA	0.1194	V	0.1203		Q	0.1193		S	0.0637	
										0.2418
		EA	0.1203		EA	0.1101		C	0.1780	

Table 7.3: Results of splitting 2 and 3 objectives

P	Atts	Wts	Atts	Wts		Atts	Wts		Atts	Wts		Atts	Wts	
1)	E	0.7188	T	0.3926		T	0.3927		E	0.5586		T	0.3651	
					0.7589			0.7925						0.7302
	V	0.0579	A	0.3663		A	0.3998		F	0.0667		A	0.2651	
											0.1106			
	EA	0.2234	F	0.0650		V	0.0369		Q	0.0439		F	0.437	
					0.1043									0.0788
			Q	0.0393		S	0.0660		S	0.1984		Q	0.351	
								0.1707			0.3308			
			EA	0.1368		C	0.1047		C	0.1324		S	0.780	
														0.1947
												C	0.1167	
2)	E	0.7306	T	0.4045		T	0.4680		E	0.5852		T	0.4335	
					0.7956			0.7880						0.7510
	V	0.0810	A	0.3311		A	0.3200		F	0.0435		A	0.3175	
											0.1385			
	EA	0.1884	V	0.0341		V	0.0424		Q	0.0950		F	0.0297	
					0.1050									0.0881
			Q	0.0709		S	0.0848		S	0.1312		Q	0.0584	
								0.1696			0.2694			
			EA	0.0994		C	0.0848		C	0.1382		S	0.0804	
														0.1608
												C	0.0804	
3)	E	0.6738	T	0.4028		T	0.3928		E	0.5395		T	0.3770	
					0.7536			0.7063						0.6740
	V	0.2255	A	0.3508		A	0.3135		F	0.1427		A	0.2970	
											0.3515			
	EA	0.1007	V	0.0793		V	0.1826		Q	0.2088		F	0.1004	
					0.1872									0.2405
			Q	0.1079		S	0.0773		S	0.0757		Q	0.1401	
								0.1112			0.1091			
			EA	0.0592		C	0.0339		C	0.0334		S	0.0575	
														0.0856
												C	0.0281	
4)	E	0.1462	T	0.0693		T	0.0810		E	0.1641		T	0.0713	
					0.1891			0.4862						0.2846
	V	0.6569	A	0.1198		A	0.4052		F	0.3998		A	0.2133	
											0.6554			
	EA	0.1963	V	0.3962		V	0.3389		Q	0.2556		F	0.3666	
					0.6542									0.5730
			Q	0.2580		S	0.0899		S	0.0902		Q	0.2064	
								0.1798			0.1804			
			EA	0.1568		C	0.0899		C	0.0902		S	0.0713	
														0.1426
												C	0.0713	
5)	E	0.6833	T	0.3266		T	0.3323		E	0.5421		T	0.3266	
					0.7173			0.7577						0.6828
	V	0.1168	A	0.3907		A	0.4254		F	0.0651		A	0.3662	
											0.1735			
	EA	0.1998	V	0.1001		V	0.0895		Q	0.1084		F	0.0500	
					0.2304									0.1297
			Q	0.1303		S	0.1005		S	0.1909		Q	0.0797	
								0.1529			0.2845			

			EA	0.0523		C	0.0524		C	0.0936		S	0.1228	
														0.1875
												C	0.0647	
6)	E	0.7440	T	0.5368		T	0.4531		E	0.5176		T	0.4189	
					0.6642			0.6242						0.5661
	V	0.1336	A	0.1274		A	0.1711		F	0.1285		A	0.1472	
											0.2570			
	EA	0.1194	V	0.1131		V	0.1504		Q	0.1285		F	0.1197	
					0.2262									0.2394
			Q	0.1131		S	0.0630		S	0.1127		Q	0.1197	
								0.2255			0.2252			
			EA	0.1096		C	0.1625		C	0.1125		S	0.0792	
														0.1945
												C	0.1153	

7.4.2 Discussion: Table 7.3 (Case of splitting two and three objectives)

It is observed from the results that in 81% cases, the sum of the weights of the splitted attributes is more that the weight of the corresponding unsplitted objective. When the most and the second most important objectives are subdivided, there is greater possibility of decreasing weight of the second most important objective (62%). When the least important (here second and third for the Persons 1, 2, 3, 5, and 6) objectives are splitted, their weights always increase. This is quite evident from the column representing weights due to (E, F, Q, S, C).

In the case of splitting all the three objectives simultaneously, the sum of weights of the most important attributes (here teaching and academic environment) is decreased in most cases and this is quite expected, because splitting of a lesser important objective increases its weight diminishing the weights of more important objectives.

7.5 A Statistical Test of Overweighting Bias

The overweighting bias has been tested statistically by performing an analysis of variance in asymmetrical factorial design. There are two factors ‘round’ and ‘objectives’. The levels of ‘round’ are *non-split* and *split* and those of ‘objectives’ are O_1, O_2, and O_3. If there is any overweighting bias, then that will be manifested only by the interaction effect. Table 7.4 gives the F-ratios for the interaction terms. Four F-ratios, which correspond to only single objective splitting case and the splitting of the lesser important objectives, are significant at 5% level of significance. In the other three cases, the bias is not significant.

7.6 Effect of Splitting Objectives on the Ranking of Alternatives

In the school selection problem, there are 3 schools from which the best one should be chosen. Each school achieves certain attribute up to certain level. For this reason, the performance of each school against each of the unsplitted as well as splitted attributes has been kept fixed for all persons’ evaluations. The performances of the schools is shown in Table 7.5. Local weights of the schools with respect to the attributes are shown in Table 7.6.

According to Saaty’s (1986a) fourth axiom of AHP, the inherent or actual ranking would be obtained if all the concerned attributes are incorporated in the hierarchy. It is a recognized fact that, while determining ranking of a set of alternatives, one can ignore those criteria which have negligible impact on the overall weights. The present work verifies this assertion experimentally. The overall ranking of the schools based upon six persons' (taking one from each category) judgments are shown in Table 7.7.

Table 7.4: F-ratios for interaction terms between first and second round weights for objectives

Attributes	F-ratios
(T,A,V,EA)	6.8668**
(E,F,Q,EA)	6.4113**
(E,V,S,C)	5.1030*
(T,A,F,Q,EA)	1.71667
(T,A,V,S,C)	1.000
(E,F,Q,S,C)	12,1977**
(T,A,F,Q,S,C)	0.1834

** significant at 1% level
*   significant at 5% level

Table 7.5: Performances of the schools

E*	X	Y	Z	V	X	Y	Z	EA	X	Y	Z
X	1	1	4	X	1	1/2	1/3	X	1	1/2	1/6
Y	1	1	3	Y	2	1	1/2	Y	2	1	1/4
Z	1/4	1/3	1	Z	3	2	1	Z	6	4	1
T	X	Y	Z	A	X	Y	Z	F	X	Y	Z
X	1	2	3	X	1	1/2	3	X	1	1/2	1/4
Y	1/2	1	3	Y	2	1	5	Y	2	1	1/2
Z	1/5	13	1	Z	1/3	1/5	1	Z	4	2	1
Q	X	Y	Z	S	X	Y	Z	C	X	Y	Z
X	1	1/3	1/4	X	1	1/3	1/8	X	1	1/2	1/5
Y	3	1	1/2	Y	3	1	1/2	Y	2	1	1/3
Z	4	2	1	Z	8	2	1	Z	5	3	1

**The letter in the left-top corner of each matrix represents the criterion on which the comparisons have been made.*

Table 7.6: Priority weights of the three schools on various criteria

Criteria	X	Y	Z
E	0.4579	0.4160	0.1261
V	0.1635	0.2969	0.5395
EA	0.1062	0.1929	0.7009
T	0.5815	0.3090	0.095
A	0.3090	0.5815	0.1095
F	0.1429	0.2857	0.5714
Q	0.1220	0.3196	0.5584
S	0.0752	0.1830	0.7418
C	0.1220	0.2298	0.6482

Table 7.7: Ranking of the schools for various cases

1.

At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
E	0.7183	X	0.3621	T	0.4180	X	0.3845	E	0.6342	X	0.3340	E	0.6493	X	0.3337
V	0.0579	Y	0.3591	A	0.3801	Y	0.3938	F	0.1053	Y	0.3511	V	0.0529	Y	0.3457
EA	0.2234	Z	0.2788	V	0.0444	Z	0.2217	Q	0.0550	Z	0.3149	S	0.1802	Z	0.3205
				EA	0.1575			EA	0.2056			C	0.1149		
Rank:	X,Y,Z				Y,X,Z				Y,X,Z				Y,X,Z		
At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
T	0.3926	X	0.3701	T	0.3927	X	0.3757	E	0.5584	X	0.3017	T	0.3651	X	0.3540
A	0.3663	Y	0.3918	A	0.3998	Y	0.4009	F	0.0667	Y	0.3322	A	0.3610	Y	0.3829
F	0.0650	Z	0.2381	V	0.0369	Z	0.2234	Q	0.0439	Z	0.3661	F	0.0437	Z	0.2631
Q	0.0393			S	0.0660			S	0.1984			Q	0.0351		
EA	0.1368			C	0.1047			C	0.1324			S	0.0780		
												C	0.1167		
Rank:	Y,X,Z				Y,X,Z				Z,Y,Z				Y,X,Z		

2.

At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
E	0.7306	X	0.3561	T	0.5030	X	0.3878	E	0.6658	X	0.3345	E	0.6347	X	0.3192
V	0.0810	Y	0.3667	A	0.3273	Y	0.4029	F	0.0501	Y	0.3655	V	0.0624	Y	0.3493
EA	0.1884	Z	0.2772	V	0.0530	Z	0.2092	Q	0.1193	Z	0.3000	S	0.1514	Z	0.3315
				EA	0.1105			EA	0.1648			C	0.1514		
Rank:	Y,X,Z				Y,X,Z				Y,X,Z				Y,Z,X		
At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
T	0.4645	X	0.3734	T	0.4680	X	0.3651	E	0.5852	X	0.3027	T	0.4335	X	0.3502
A	0.3311	Y	0.3970	A	0.3200	Y	0.3798	F	0.0435	Y	0.3535	A	0.3175	Y	0.3788
F	0.0341	Z	0.2296	V	0.0424	Z	0.2551	Q	0.0950	Z	0.3439	F	0.0297	Z	0.2710
Q	0.0709			S	0.0848			S	0.1382			Q	0.0584		
EA	0.1994			C	0.0848			C	0.1382			S	0.0884		
												C	0.0884		
Rank:	Y,X,Z				Y,X,Z				Y,Z,X				Y,X,Z		

3.

At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
E	0.6738	X	0.3678	T	0.4138	X	0.4176	E	0.6295	X	0.3441	E	0.5674	X	0.3307
V	0.2255	Y	0.3643	A	0.3795	Y	0.3847	F	0.1156	Y	0.3612	V	0.2822	Y	0.3451
EA	0.1007	Z	0.2679	V	0.1398	Z	0.1977	Q	0.1691	Z	0.2947	S	0.1089	Z	0.3243
				EA	0.0668			EA	0.0858			C	0.0414		
Rank:	X,Y,Z				X,Y,Z				Y,Z,X				Y,X,Z		
At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
T	0.4228	X	0.3965	T	0.3928	X	0.3947	E	0.5395	X	0.3130	T	0.3770	X	0.3774
A	0.3508	Y	0.3876	A	0.3135	Y	0.3783	F	0.1427	Y	0.3433	A	0.2970	Y	0.3765
F	0.0893	Z	0.2159	V	0.1826	Z	0.2270	Q	0.2088	Z	0.3437	F	0.1004	Z	0.2461
Q	0.1079			S	0.0773			S	0.0757			Q	0.1401		
EA	0.0592			C	0.0339			C	0.0334			S	0.0575		
												C	0.0281		

Rank: X,Y,Z	X,Y,Z	Z,Y,X	X,Y,Z

4.

At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
E	0.1462	X	0.1952	T	0.0622	X	0.2003	E	0.0902	X	0.1602	E	0.2763	X	0.2295
V	0.6569	Y	0.2937	A	0.1630	Y	0.3207	F	0.4753	Y	0.2963	V	0.4873	Y	0.3084
EA	0.1963	Z	0.5111	V	0.5501	Z	0.4790	Q	0.3088	Z	0.5435	S	0.1182	Z	0.4621
				EA	0.2247			EA	0.1257			C	0.1182		
Rank:	Z,Y,X			Z,Y,X				Z,Y,X				Z,Y,X			
At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
T	0.0693	X	0.1821	T	0.0810	X	0.2446	E	0.1641	X	0.1812	T	0.0713	X	0.1990
A	0.1198	Y	0.3170	A	0.4052	Y	0.3969	F	0.3998	Y	0.2415	A	0.2133	Y	0.3441
F	0.3962	Z	0.5010	V	0.3339	Z	0.3585	Q	0.2556	Z	0.5773	F	0.1366	Z	0.4569
Q	0.2580			S	0.0899			S	0.0902			Q	0.2064		
EA	0.1568			C	0.0899			C	0.0902			S	0.0713		
												C	0.0713		
Rank:	Z,Y,X			Y,Z,X				Z,Y,X				Z,Y,X			

5.

At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
E	0.6833	X	0.3332	T	0.3172	X	0.3628	E	0.5673	X	0.3146	E	0.6060	X	0.3222
V	0.1168	Y	0.3575	A	0.5005	Y	0.4321	F	0.1547	Y	0.3595	V	0.1338	Y	0.3427
EA	0.1998	Z	0.2893	V	0.0754	Z	0.2052	Q	0.2032	Z	0.3259	S	0.1913	Z	0.3352
				EA	0.1069			EA	0.0747			C	0.0689		
Rank:	Y,X,Z			Y,X,Z				Y,Z,X				Y,Z,X			
At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
T	0.3266	X	0.3464	T	0.3323	X	0.3533	E	0.5421	X	0.2965	T	0.3266	X	0.3340
A	0.3907	Y	0.4084	A	0.4254	Y	0.4071	F	0.0651	Y	0.3352	A	0.3562	Y	0.3832
F	0.1001	Z	0.2452	V	0.0895	Z	0.2397	Q	0.1084	Z	0.3683	F	0.0500	Z	0.2828
Q	0.1303			S	0.1005			S	0.1909			Q	0.0797		
EA	0.0523			C	0.0524			C	0.0936			S	0.1228		
												C	0.0647		
Rank:	Y,X,Z			Y,X,Z				Z,Y,X				Y,X,Z			

6.

At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
E	0.7470	X	0.3766	T	0.6139	X	0.4344	E	0.6513	X	0.3415	E	0.5870	X	0.3233
V	0.1336	Y	0.3735	A	0.1455	Y	0.3332	F	0.1193	Y	0.3644	V	0.1712	Y	0.3476
EA	0.1194	Z	0.2500	V	0.1203	Z	0.2324	Q	0.1193	Z	0.2941	S	0.0637	Z	0.3291
				EA	0.1203			EA	0.1101			C	0.1781		
Rank:	X,Y,Z			X,Y,Z				Y,X,Z				Y,Z,X			
At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.	At.	Wt.	Alt.	Wt.
T	0.5368	X	0.3931	T	0.4531	X	0.3655	E	0.5176	X	0.2932	T	0.4189	X	0.3408
A	0.1274	Y	0.3296	A	0.1711	Y	0.3330	F	0.1285	Y	0.3396	A	0.1472	Y	0.3285
F	0.1131	Z	0.2773	V	0.1504	Z	0.3015	Q	0.1285	Z	0.3672	F	0.1197	Z	0.3307
Q	0.1131			S	0.0630			S	0.1127			Q	0.1197		
EA	0.1096			C	0.1625			C	0.1125			S	0.0792		
												C	0.1153		
Rank:	X,Y,Z			X,Y,Z				Z,Y,X				X,Z,Y			

7.6.1 Illustration of Table 7.7

Table 7.7 shows ranking of the schools for six persons' judgments. There are eight blocks for each person. Blocks 1 and 8 correspond to the cases of non-splitting and splitting of all the objectives. Blocks 2, 3, and 4 correspond to the splitting of one objective and blocks 5, 6, 7 correspond to the splitting of two objectives at a time.

The ranking of the schools has been provided at the bottom most row of each block. According to the forth axiom of AHP, the inherent ranking is provided at the last block (i.e., for T, A, F, Q, S, C). The ranking corresponding to the penultimate block (E, F, Q, S, C) for persons 1, 2, 3, 5, 6 and that corresponding to block (T, A, V, S, C) for Person 4 are always wrong because of the subdivision of the lesser important objectives, which causes greater weights for them thereby leading to wrong ranking. It is clear from the table that the ranking corresponding to the Blocks 2, 5, 6, for Persons 1, 2, 3, 5, 6 and Blocks 3, 5, 7 for Person 4, where the most important objective(s) has been splitted, is same as the inherent ranking. In any other case, ranking is not reliable.

7.7 Concluding Remarks

The subjects displayed a near universal bias to overweight the splitted attributes (single objective splitting case). This bias is more for lesser important objectives. Splitting the least important objective always increases its weight. When the most and the second most important objectives are subdivided, there is a greater possibility of decrease of weight for the second most important objective. When all the objectives are splitted, the most important objective's weight gets decreased in most of the cases. But only the differences of weights for the single objective splitting case and the splitting of lesser important objectives are statistically significant.

While splitting of objectives has impact on their weights, naturally, it has also impact on the ranking of the alternatives. In general, non-splitting and splitting of the lesser important objectives lead to two different sets of rankings. It is well known in selection of the most preferred alternative from several ones based upon multiple objectives, that one can ignore the lesser important objective, which have obvious less impact on the overall weights of the alternatives, i.e., one can consider only the important objectives. But even in that case, if the difference between weights of the most preferred alternative and the second most preferred alternative is not significant, (i.e., almost equal), then the decision maker can simply split the most important objective(s) into two (or three) attributes (provided the splitting is meaningful) and recalculate the global weights of the alternatives. The conclusions have been drawn from a single MCDM experiment (school selection) consisting only three objectives and three alternatives. These can be verified by replicating the experiment on problems of different sizes. Further, the conclusion on the ranking of the schools can be verified by varying the performance matrices of the schools for different persons.