Chapter 6

Clusterization of Alternatives in the Analytic Hierarchy Process

6.1 Introduction

There are several methodologies such as PROMETHEE (Brans et al., 1986), MAUT (Keeny and Raiffa, 1976), ELECTRE (Roy and Vincke, 1981), LINMAP (Srivinasan and Shocker, 1973), AHP (Saaty, 1977a), TOPSIS (Hwang and Yoon, 1981) for solving discrete MCDM problems. The AHP is perhaps the most popular among them because of the following reasons:

- its ability to handle inconsistency in judgments,
- its ability to incorporate intangible or non-quantifiable criteria in the decision-making process, and
- its ease of use.

Moreover, it has been made easier to use by the microcomputer software package *Expert Choice* (Forman et al., 1983).

Despite its advantages as a powerful MCDM methodology, a major drawback in the use of AHP is the amount of work required to make all the necessary pairwise comparisons. For example, if we have a problem of determining overall weights of 10 alternatives with respect of 5 criteria, then a total of 235 comparisons must be made. In realistic situations, this number may be quite large. Harker (1987a, 1987b) presented two possible remedies regarding the curtailment of labour in filling up pairwise comparisons matrices. But while reducing comparisons, his incomplete pairwise comparison technique, in turn, invokes computation of derivatives which is also arduous for large size problems. Weiss and Rao (1987) presented a balanced incomplete block design (BIBD) technique in order to reduce the number of comparisons. But this approach too is not practicable because of its inherent complexity.

In this connection, Saaty's (1990b) suggestion of clustering alternatives into groups according to a common attribute appears to be more appropriate. In Section 6.2, we have described the idea of clusterization in an algorithmic form. In order to show the applicability of the clustering procedure, we have considered the problem of *choice of best transport aircraft* from several ones based upon a number of criteria. The problem has been solved both by the traditional AHP and the clustering procedure. We call the clustering procedure as *Clustered AHP*. The results of the two methodologies are compared and are illustrated in Section 6.3. From AHP literature, we have considered several problems (Arbel, 1983; Saaty, 1979b; Saaty and Gholamnezhad, 1982; Sinuany-Stern, 1988; Vachnadze and Markozashvili, 1987) and solved them by the clustering procedure. The results are compared with the actual results in Section 6.4. Concluding remarks are provided in Section 6.5.

6.2 Procedure for Clusterization of Alternatives

The need to reduce the number of pairwise comparisons is two-fold:

- to minimize labour and consequently time while constructing pairwise comparison matrices, and
- to obtain greater consistency

To reduce the labour in filling up all entries in the pairwise comparison matrices, Saaty (1977a, 1990b) suggested two procedures to decompose the whole set of alternatives into clusters. In his first clusterization procedure, at the most seven homogeneous elements may belong to each cluster. Then priorities are determined for each cluster by constructing PCM considering each cluster as a single element. Next, one finds out the priorities of the elements belonging to each cluster. The required priorities of the elements are obtained on multiplication of their priorities by the priority to which they belong.

In this chapter, we have used the second procedure which is valid only when the number of alternatives is quite large (>7), and evaluation scores of the alternatives with respect to certain criterion are widely dispersed. Some elementary results on the second clusterization procedure have also been developed. We present this procedure in an algorithmic form:

Step 1 *Construct the decision hierarchy of inter-related decision elements by breaking down the decision problem at hand.*

Step 2 *Calculate the weights of the criteria by the usual AHP.*

Step 3 *Adopting some (subjective) suitable scale, obtain absolute ranking of all the alternatives with respect to some criterion.*

Step 4 *Make cluster of alternatives having closer absolute scores with respect to certain criterion. Each cluster consists of at most seven elements. The smallest element of the largest cluster is included as the largest element of the next cluster and so on.*

Step 5 *Find the priority weights of all the alternatives belonging to each cluster. To pull together and make commensurate of the weights of the alternatives, divide the relating weights of all the alternatives in the second cluster by the weight of the common alternative and then multiply by its weight in the first cluster. Repeat the process for the remaining clusters. Steps 3, 4, and 5 are to be repeated for all the criteria.*

Step 6 *Obtain global ranking of all the alternatives by applying the principle of hierarchical composition.*

Theorem 6.1 *In the clustering procedure, the maximum number of pairwise comparisons necessary for n ($n \geq 7$) number of alternatives is*

$$(7/2)(n-1) - (1/2)p(6-p),$$

assuming that not more than seven elements are compared simultaneously and p ($0 \leq p \leq 6$) is the number of elements in the last cluster.

Proof: Let $\xi_1, \xi_2, \ldots, \xi_m$ be the m clusters obtained by decomposing the n alternatives. The mth cluster consists of only p alternatives, where ($0 \leq p \leq 6$). The first cluster contains seven elements and each of the remaining clusters contains six elements. One alternative from the first cluster is to be added to the second cluster, one from the second cluster to be added to the third cluster, and so on, thereby making sizes of the pairwise comparison matrices seven (except the last one, where ($0 \leq p < 6$). Then the number of clusters consisting of exactly six elements is equal to m - 2 which implies

$$7 + 6\,(m-2) + p = n, \text{ or, } m-2 = (n-7-p)/6$$

Hence, the maximum number of pairwise comparisons necessary is

$$((n-7-p)/6 + 1) \times 7(7-1)/2 + (p+1)p/2$$
$$= (7/2)(n-1) - (1/2)p(6-p).$$

Corollary 6.1 *The number of pairwise comparisons saved is at least*

$$(1/2)\,(n-1)\,(n-7) + (1/2)p(6-p)$$

where n(≥ 7) is the total number of alternatives and the last cluster contains only p ($0 \leq p \leq 6$) elements.

Proof: By Theorem 6.1, the maximum number of comparisons necessary is

$$(7/2)\,(n-1) - (1/2)p(6-p).$$

Without clustering, the total number of comparisons required for n alternatives is equal to n(n-1)/2. Therefore, the number of comparisons saved is at least

$$n\,(n-1)/2 - (7/2)\,(n-1) + (1/2)p(6-p)$$
$$= (1/2)\,(n-1)\,(n-7) + (1/2)p(6-p).$$

Saaty (1977a) defined the ratio of the number of direct pairwise comparisons to the total number of comparisons in the clustering procedure, as the *efficiency of the hierarchy*.

Corollary 6.2: *The efficiency of a hierarchy is of order n/7.*

Proof: The number of direct pairwise comparisons for n alternatives is equal to n(n-1)/2. By theorem 6.1, after clusterization, the maximum number of comparisons required is

$$(7/2)\,(n-1) - (1/2)p(6-p),\ 0 \leq p \leq 6.$$

Therefore, the efficiency of the hierarchy

$$= (1/2)n(n-1)/((7/2)n-1) - (1/2)p(6-p))$$
$$> (1/2)n(n-1)/((7/2)n-1)$$
$$= n/7.$$

Remark 6.1: The maximum number of elements in a cluster has been kept seven because in general, one can compare 7 ± 2 elements simultaneously without any confusion (Miller, 1956). Also it has been noticed that *"using the consistency index C* *the number 7 is a good practical bound on n, at last outpost, as far as consistency is concerned"* (Saaty, 1977a, page 275).

Remark 6.2: The corollary 6.2 has been proved by Saaty (1977a) for the first clusterization procedure.

6.3 Choice of the Best Transport Aircraft

The complex airlift problem has been investigated by many military researchers (e.g., Quade, 1978). Defence choices are difficult due to the presence of multiple conflicting objectives, many decision makers, non-availability of market mechanism to determine the relationship between a proposed system's cost and its military worth of effectiveness, etc. (Ng, 1980).

Any defence decision calls for mediation over political, social, and economic life of citizens. Transport aircraft choice is not an exception. Particular choice of transport aircraft may have an impact on domestic employment, regional economic activity, technology transfer, etc. So, at the time of choice of the aircraft, the following gross criteria are of utmost importance from defence point of view:

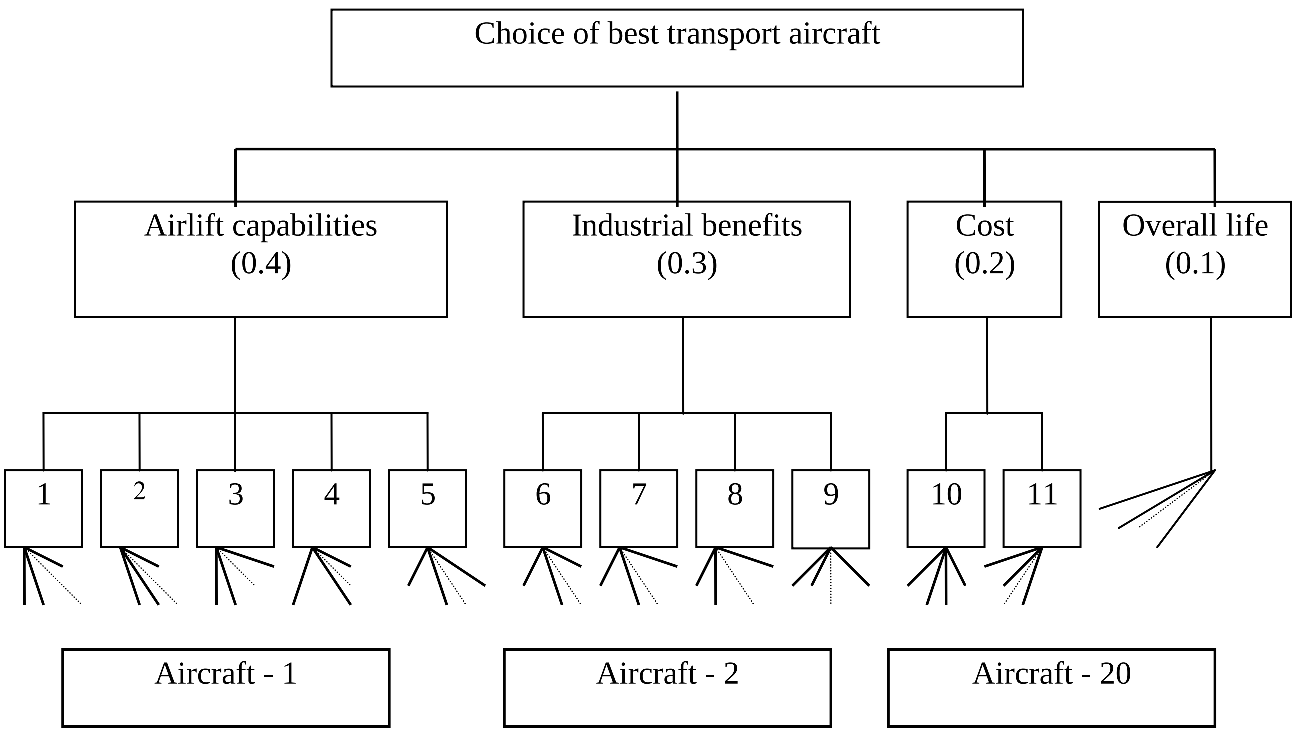
(i) satisfaction of military requirements,
(ii) maximization of industrial benefits,
(iii) cost, and
(iv) overall life of the aircraft.

To be more specific, each of the foregoing criteria can be splitted into several sub-criteria. For example, the criterion 'satisfaction of military requirements' can be divided into five sub-criteria: a) airlift requirements; b) aircraft characteristics; c) route structure; d) airbases and their support characteristics; and e) system policies. A comprehensive description of the criteria and sub-criteria is available in Ng (1980).

Thus, it is clear that the transport aircraft choice problem is characterized by a multitude of incommensurable criteria. Fig. 6.1 depicts the salient factors of the problems in a hierarchical form.

From Indian perspective, the weights of the criteria and sub-criteria have been determined in consultations with a number of Indian airforce officials. After calculating the weights of the criteria and sub-criteria (weights are provided in Fig. 6.1 itself), the relative standing of 20 types of transport aircraft with respect to one criterion at a time has been determined. Table 6.1 shows one comparison matrix for the alternatives constructed for the criterion 'aircraft characteristics'. There are altogether 12 such 20 x 20 matrices for 12 criteria and sub-criteria. The decomposed matrices obtained from the matrix in Table 6.1 are shown in Table 6.2. After determination of the weights of all the alternatives by traditional AHP, the same have been determined using the algorithm described in Section 6.2. The procedure is repeated for all the criteria. The results are shown in Table 6.3.

Remarks 6.3: By a series of consultations with some officials working in Indian Air Force, the data for the pairwise comparison matrices are obtained. The entries in the pairwise comparison matrices depend upon the number of alternatives, i.e., deletion of some alternative(s) from the set of alternatives may alter the strength of preference of any alternative over another. Presumably, the difference of the two preference ratios will not be significantly high. That is why we have retained the same preference ratios in the decomposed matrices. Clearly, the weights can also be determined by taking different judgments in the decomposed matrices.



1. Aircraft requirements (0.25)
2. Aircraft characteristics (0.4)
3. Airbase and support characteristics (0.4)
4. Route structure (0.1)
5. System policies (0.15)
6. Distribution of works among industries (0.1)
7. Creation of jobs (0.15)
8. Transfer of technology (0.6)
9. Regional distribution of industrial activities (0.15)
10. Initial cost (0.7)
11. Operating cost (0.3)

Fig. 6.1: Hierarchy for the transport aircraft choice problem

Table 6.1: One pairwise comparison matrix for the twenty alternatives

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	9	4	2	4	5	3	2	1	1	3	7	3	2	2	1	1	2	3	6
2		1	1/2	1/5	1/3	1/2	1/3	1/5	1/9	1/5	1/3	1	1/3	1/6	1/6	1/7	1/7	1/4	1/3	1/2
3			1	1/2	1	2	1	1/2	1/4	1/2	1	2	1	1/2	1/2	1/3	1/3	1	1	2
4				1	2	2	2	1	1/2	1	2	4	2	1	1	1/2	1/2	1	2	3
5					1	1	1	1/2	1/4	1/2	1	2	4	1/2	1/2	1/3	1/3	1/2	1	2
6						1	1/2	1/3	1/5	1/3	1/2	2	1/2	1/3	1/3	1/4	1/4	1/2	1/2	1
7							1	1/2	1/4	1/2	1	3	1	1/2	1/2	1/3	1/3	1	1	2
8								1	1/2	1	2	4	2	1	1	1	1	1	2	3
9									1	2	4	8	4	2	22	22	2	3	3	6
10										1	2	5	2	1	1	1/2	1/2	2	2	4
11											1	2	1	1/2	1/2	1/3	1/3	1/2	1	2
12												1	1/2	1/5	1/5	1/6	1/6	1/3	1/3	1
13													1	1/2	1/2	1/3	1/3	1	1	2
14														1	1	1/2	1/2	2	2	4
15															1	1/2	1/2	2	2	4
16																1	2	3	4	5
17																	1	2	3	5
18																		1	2	3
19																			1	2
20																				1

Now the global ranking of all the alternatives is obtained for both the methods. At first we have obtained the ranking for 4 attributes only. Then the process is repeated adding one attribute at a time. Each time Pearson's rank correlation coefficient is calculated.

The detailed results are shown in Table 6.4. It is worth noting that, in all the cases, the rank correlation coefficients are greater than 0.9.

For the present transport aircraft problem, to compare all the alternative with respect to all the 12 attributes, Saaty's (1977a) AHP takes 2280 pairwise comparisons, whereas the Clustered AHP takes only 683 comparisons (thereby saving 1597 comparisons). It is to be noted that the sum of the pairwise comparisons for the decomposed matrices (of sizes less or equal to 7) with respect to certain attribute may be different from that for the decomposed matrices with respect to any other attribute.

Table 6.2: Decomposed matrices

	1	9	16	17
1	1	1	1	1
9		1	2	2
16			1	1
17				1

	16	4	8	10	14	15
16	2	2	2	2	2	2
4		1	1	1	1	1
8			1	1	1	1
10				1	1	1
14					1	1
15						1

	4	3	7	11	13	18	19
4	1	2	2	2	2	1	2
3		1	1	1	1	1	1
7			1	1	1	1	1
11				1	1	1/2	1
13					1	1	1
18						1	2
19							1

	19	2	5	6	12	20
19	1	3	1	2	3	2
2		1	1/3	1/2	1	½
5			1	1	2	2
6				1	2	1
12					1	1
20						1

Following Freund (1992), a statistical test has been performed for the rank correlation coefficients to test the variability of the rankings obtained by the two procedures (Saaty's AHP and Clustered AHP). Let

H_0 : the two rankings are significantly different

H_1 : the two rankings are not significantly different

At 1% level of significance, the value of the statistic z = 2.575. The computed values of z = $r\sqrt{n-1}$ for various number of attributes are shown in Table 6.5. From this table, we observe that the computed z-value does not fall within the critical region for any number of attributes. So, we must reject the null hypothesis H_0. Therefore, the two rankings are not significantly different.

6.4 Some Selected Problems from the AHP Literature

Apart from the aircraft choice problem, we have also considered five other problems from the published AHP literature. To show the applicability of the clusterization procedure, we have solved these problems by it and compared the results with the actual ones.

Problem 6.1: High level nuclear waste management problem

In a nuclear industry, safe disposal of 'high level' waste, some of which remain radio-active for hundreds of thousands of years, is a burning problem. Because of toxicity and long half-lives, management of such nuclear waste is the most challenging problem in radio-active management. Saaty and Gholamnezhad (1982) made a thorough discussion of this problem. They considered the following five options by which disposal can be done:

(a) geologic disposal using conventional mining techniques,
(b) very deep hole,
(c) island disposal,
(d) subseabed disposal, and
(e) disposal into space.

Table 6.3: Priority weights of 20 alternatives by Method 1 and Method 2

	Attribute 1		Attribute 2		Attribute 3		Attribute 3		Attribute 5		Attribute 6	
Alt.	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2
1	0.1013	0.1237	0.1013	0.0085	0.1384	0.1551	0.0341	0.0345	0.0137	0.0085	0.0153	0.0101
2	0.0102	0.0077	0.1423	0.1629	0.0169	0.0145	0.0341	0.0345	0.0331	0.0285	0.1032	0.1010
3	0.0308	0.0283	0.0670	0.0536	0.0711	0.0775	0.0247	0.0295	0.0652	0.0599	0.1443	0.2019
4	0.0554	0.0511	0.0270	0.0239	0.0346	0.0300	0.0341	0.0345	0.0598	0.0599	0.0299	0.0215
5	0.0291	0.0214	0.0670	0.0536	0.1125	0.1030	0.0264	0.0295	0.1004	0.1101	0.0299	0.0215
6	0.0214	0.0159	0.0271	0.0239	0.0273	0.0214	0.0122	0.0126	0.1035	0.1387	0.0100	0.0050
7	0.0324	0.0283	0.0122	0.0101	0.0132	0.0102	0.0354	0.0345	0.1258	0.1748	0.0376	0.0239
8	0.0565	0.0511	0.1040	0.1027	0.0549	0.0515	0.0305	0.0345	0.0747	0.0676	0.0673	0.0642
9	0.1214	0.1740	0.0353	0.0268	0.0840	0.0877	0.0178	0.0181	0.0409	0.0376	0.0147	0.0101
10	0.0627	0.0511	0.1754	0.2587	0.0840	0.0877	0.0763	0.0689	0.0174	0.0128	0.0444	0.0341
11	0.0306	0.0256	0.0288	0.0268	0.0840	0.0877	0.0763	0.0689	0.0137	0.0085	0.0309	0.0194
12	0.0134	0.0094	0.0073	0.0045	0.0234	0.0174	0.0609	0.0689	0.0202	0.0152	0.0969	0.1010
13	0.0317	0.0283	0.0073	0.0045	0.0234	0.0174	0.0939	0.0782	0.0681	0.0599	0.0157	0.0101
14	0.0607	0.0511	0.0782	0.0686	0.0118	0.0079	0.1112	0.1112	0.0197	0.0152	0.0523	0.0384
15	0.0598	0.0511	0.0165	0.0138	0.0118	0.0102	0.0948	0.0782	0.0197	0.0152	0.1000	0.1010
16	0.0948	0.1022	0.0703	0.0602	0.0226	0.0174	0.0535	0.0689	0.0376	0.0266	0.0234	0.0147
17	0.0948	0.1022	0.0349	0.0239	0.0199	0.0159	0.0545	0.0689	0.0196	0.0152	0.0149	0.0101
18	0.0440	0.0391	0.0294	0.0268	0.0753	0.0877	0.0790	0.0689	0.0303	0.0285	0.0102	0.0050
19	0.0320	0.0256	0.0491	0.0403	0.0753	0.0877	0.0201	0.0222	0.0658	0.0599	0.0102	0.0050
20	0.0167	0.0126	0.0106	0.0085	0.0155	0.0112	0.0299	0.0327	0.0658	0.0599	0.0488	0.2019

Table 6.3: (Continued)

Alt.	Attribute 7		Attribute 8		Attribute 9		Attribute 10		Attribute 11		Attribute 12	
	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2
1	0.1106	0.1172	0.0237	0.0154	0.1022	0.1186	0.0699	0.0719	0.0230	0.0179	0.0470	0.0508
2	0.0090	0.0044	0.0380	0.0308	0.1353	0.1768	0.0586	0.0719	0.0217	0.0005	0.0470	0.0508
3	0.0263	0.0201	0.0164	0.0154	0.0638	0.0593	0.0398	0.0470	0.0395	0.0358	0.0595	0.0562
4	0.0514	0.0485	0.1050	0.1122	0.0545	0.0530	0.0282	0.0260	0.0698	0.0717	0.0503	0.0562
5	0.1516	0.1516	0.0201	0.0154	0.0539	0.0530	0.0390	0.0470	0.0217	0.0090	0.0595	0.0562
6	0.0227	0.0149	0.1050	0.1122	0.0730	0.0767	0.0183	0.0130	0.0395	0.0358	0.0686	0.0648
7	0.0651	0.0622	0.0783	0.0739	0.0631	0.0593	0.0803	0.0719	0.0395	0.0402	0.0470	0.0508
8	0.0130	0.0077	0.0076	0.0044	0.0252	0.0161	0.0183	0.0130	0.0729	0.0841	0.0503	0.0508
9	0.0330	0.0243	0.0380	0.0308	0.0108	0.0063	0.0282	0.0260	0.0328	0.0320	0.0739	0.0746
10	0.0330	0.0243	0.0237	0.0154	0.0312	0.0182	0.0183	0.0103	0.0230	0.0179	0.0866	0.0746
11	0.0810	0.0865	0.0103	0.0081	0.0252	0.0161	0.0183	0.0130	0.0810	0.0841	0.0470	0.0508
12	0.0546	0.0485	0.0783	0.0739	0.0388	0.0297	0.0183	0.0130	0.0698	0.0743	0.0595	0.0562
13	0.0227	0.0149	0.0602	0.0466	0.0257	0.0161	0.1210	0.1310	0.0217	0.0090	0.0115	0.0107
14	0.1306	0.1650	0.1504	0.2245	0.0257	0.0161	0.1897	0.1826	0.0395	0.0358	0.0232	0.0254
15	0.0093	0.0044	0.0390	0.0308	0.0794	0.0884	0.0596	0.0719	0.0328	0.0320	0.0232	0.0254
16	0.0203	0.0126	0.0602	0.0466	0.0116	0.0063	0.0282	0.0260	0.1193	0.1486	0.0686	0.0646
17	0.0130	0.0077	0.0201	0.0154	0.1179	0.1534	0.0744	0.0719	0.0623	0.0608	0.0273	0.0269
18	0.0472	0.0384	0.0103	0.0081	0.0127	0.0068	0.0773	0.0719	0.0810	0.0841	0.0273	0.0269
19	0.0927	0.0970	0.0103	0.0081	0.0275	0.0161	0.0744	0.0719	0.0395	0.0464	0.0269	0.0269
20	0.0130	0.0077	0.1050	0.1122	0.0224	0.0136	0.0398	0.0470	0.0698	0.0717	0.0961	0.1002

Method 1 (M-1) = Saaty’s AHP, Method 2 (M-2) = Clustered AHP

Table 6.4: Global Ranking of Alternatives for Various no. of Attributes

	A.N.=4		A.N.=5		A.N.=6		A.N.=7		A.N.=8		A.N.=9		A.N.=10		A.N.=11		A.N.=12	
Rank	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2	M-1	M-2
1	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
2	10	10	16	10	10	10	10	10	16	10	10	10	16	10	10	10	10	10
3	2	2	10	16	16	2	16	16	10	16	16	2	10	2	2	2	10	2
4	16	16	2	2	2	16	2	2	4	4	2	16	2	16	16	16	2	16
5	4	4	4	4	4	4	4	4	7	7	4	7	7	7	7	7	4	20
6	8	20	20	20	7	9	9	9	2	2	7	4	4	4	4	20	7	4
7	20	8	7	9	1	1	8	20	20	20	6	6	6	6	20	4	20	7
8	6	6	13	6	1	7	7	1	6	6	5	20	20	20	6	6	5	1
9	7	7	17	7	13	17	20	8	5	9	220	9	5	17	8	8	1	6
10	13	13	6	17	9	20	5	7	8	5	8	1	17	5	5	17	6	17
11	3	3	9	13	8	8	17	6	9	8	17	8	8	1	17	5	8	9
12	5	19	8	8	5	13	1	17	13	1	9	17	13	8	13	1	17	5
13	19	5	1	1	19	19	6	5	19	19	1	5	1	9	3	3	13	8
14	17	18	18	15	20	5	19	19	1	13	13	19	9	13	1	9	3	3
15	18	17	15	18	18	6	13	18	17	17	3	3	3	19	9	13	9	13
16	12	12	19	19	6	18	18	13	3	3	19	18	19	3	19	15	18	19
17	9	15	12	3	3	15	3	3	18	18	18	13	18	18	15	19	19	19
18	15	9	3	12	15	3	12	12	12	12	12	15	12	15	12	12	12	15
19	1	1	5	5	12	12	15	15	15	15	15	12	15	12	18	18	15	12
20	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
R.C.C	0.9939		0.9699		0.9548		0.9669		0.9879		0.9504		0.9834		0.9849		0.9594	

A.N. = Attribute Number, R.C.C. = Rank Correlation Coefficient

Table 6.5: z-values for various number of attributes

Number of attributes	4	5	6	7	8	9	10	11	12
Computed value of z	4.3323	4.2277	4.1618	4.2146	4.3062	4,1427	4,2866	4.2931	4.1819

These five strategies are judged on eight criteria, namely, 1) state of technology, 2) health, safety, and environmental impacts, 3) cost, 4) socio-economic impacts, 5) lead-time, 6) political consideration, 7) resource consumption, and 8) aesthetic effects.

The overall weights and ranking obtained by Saaty and Gholamnezhad (1982) and those obtained by the Clustered AHP are shown in Table 6.6. It may be noticed that both the rankings are exactly same.

Table 6.6: Overall weights and ranking of the alternatives in nuclear waste management problem

		Saaty's AHP			Clustered AHP		
Alternatives		Weights	Rank	P.C. Reqd.	Weights	Rank	P.C. Reqd.
1.	Geologic disposal	0.3000	1	162	0.3509	1	99
2.	Very deep hole	0.1720	3		0.1459	3	
3.	Island disposal	0.1580	4		0.1307	4	
4.	Subseabed disposal	0.1390	5		0.1120	5	
5.	Space disposal	0.2280	2		0.2590	2	

P.C. Reqd. = Pairwise Comparisons Required.

Problem 6.2: Ranking of sixteen sports teams

Sinuany-Stern (1988), in his paper, predicted the ranking of 16 soccer teams participating in the Israeli National League. The evaluation was based on six criteria: the facility, the coach, the players, the fans, the previous season's performance, and the current performance. The overall weights and the ranking of the teams obtained by him and those obtained by the Clustered AHP are shown in Table 6.7.

By applying the Clustered AHP, we have also solved three other problems, viz, 'a university budget allocation problem' (Arbel, 1983), 'U.S. - OPEC energy conflict' (Saaty, 1979b) and 'a relay race team formation' (Vachnadze and Markozashvili, 1987). The results are similar to those obtained by the respective authors. Details are omitted to avoid monotony.

Table 6.7: The overall weights and ranking of 16 soccer teams

	Saaty's AHP			Clustered AHP		
Teams	Weights	Rank	P.C. Reqd.	Weights	Rank	P.C. Reqd.
1	0.0507	9	645	0.0446	9	272
2	0.1138	2		0.1304	2	
3	0.0894	5		0.0841	5	
4	0.0640	7		0.665	7	
5	0.1328	1		0.1495	1	
6	0.1003	3		0.1191	3	
7	0.0664	6		0.771	6	
8	0.0269	15		0.026	14	
9	0.0446	10		0.0368	10	
10	0.0245	16		0.0193	15	
11	0.0608	8		0.0571	8	
12	0.0323	13		0.0254	13	
13	0.0956	4		0.880	4	
14	0.0284	14		0.0175	16	
15	0.0392	11		0.0349	11	
16	0.0357	12		0.0300	12	

6.5 Concluding Remarks

While applying AHP for a large-scale discrete choice problem, a large number of pairwise comparisons appear as an intriguing problem. Many suggestions are proposed to reduce the number of comparisons. But none has emerged so fruitful from the application point of view. In this connection, Saaty's (1990b) proposal of clustering alternatives seems to be a better remedy. But no work has been done to verify its applicability. In this chapter, an attempt has been made to fill up this gap based upon experiments on a series of real world problems. It has been shown that in the clustering procedure, the number of comparisons required is much less than that required in the unified approach but both the rankings are sufficiently close to each other (manifested by rank correlation coefficient and substantiated by subsequent statistical test).